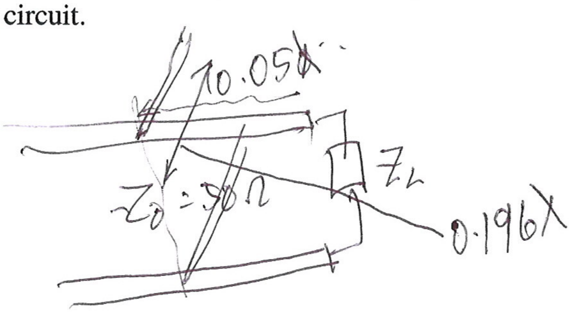
Part A

[A.] A Si-Ge heterojunction bipolar transistor is used in an amplifier circuit at 2GHz. The equivalent circuit of the input base for a grounded emitter configuration is estimated to have an input resistance of 5Ω in parallel with a capacitance of 1pF. Design a single stub matching circuit to a 50Ω line. Use a Smith chart to design your matching circuit.



$$Z_L = \frac{1}{\frac{1}{5} + j2\pi \times 2\times 10^{9} \times 10^{-12}}$$

$$Y_L = 0.2 + j0.0126\ S.$$

$$y_L = Y_L \times 50 = 10 + j0.628$$

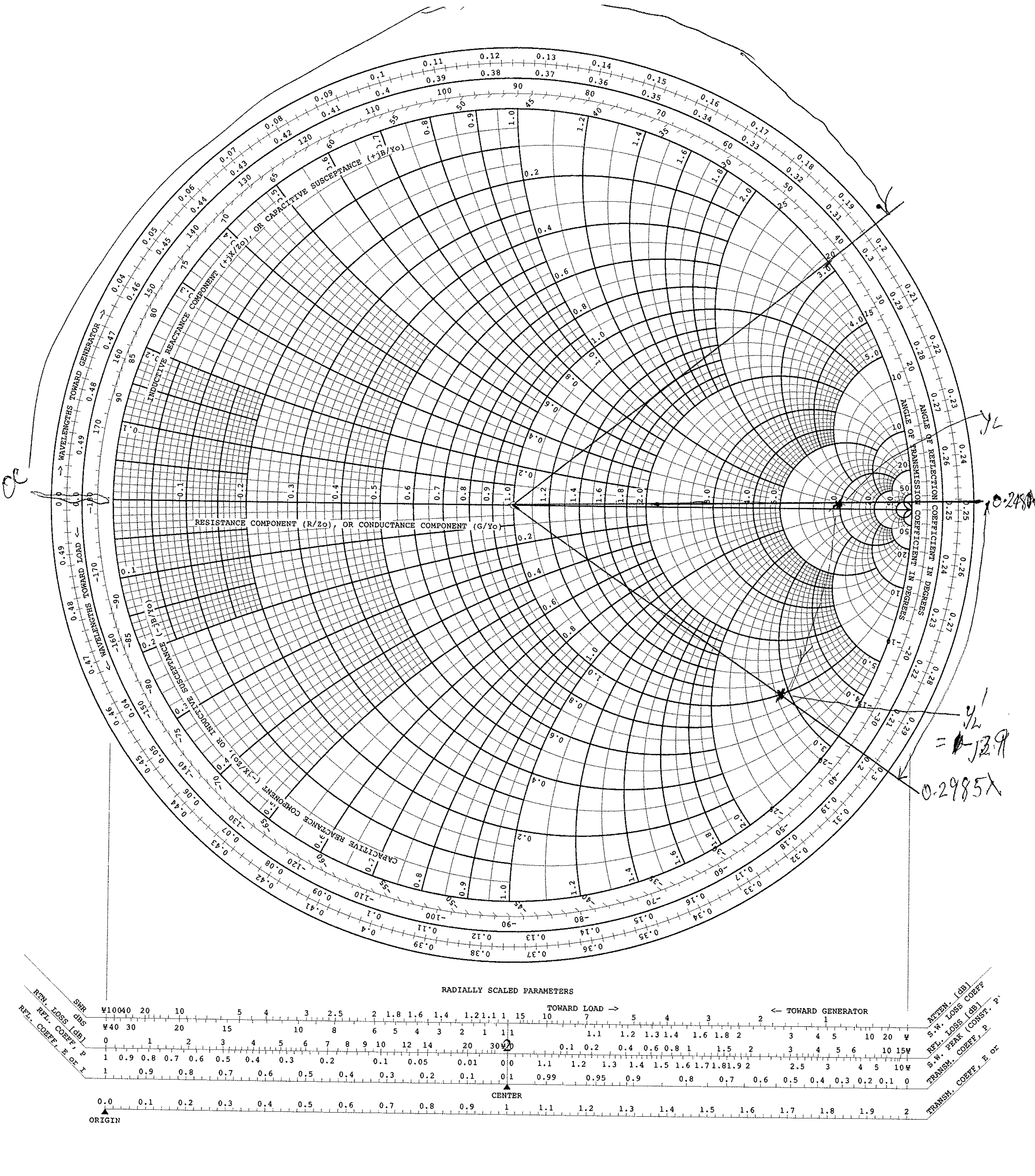
Rotating clockwise, towards generator; the circle cuts the $g = 1.0$ to give

$$y_L' = 1.0 - j2.9\ S$$

$$\text{distance} = \begin{array}{r} 0.2985\lambda \\ -\,0.2481 \\ \hline 0.0504\lambda \end{array} \quad -50\Omega$$

Stub ~~impe~~ susceptance $= j2.9\,S$

With open circuit stub, stub length at 50Ω.
0.1962λ

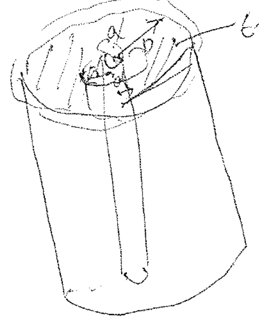

WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD ->
<- TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN
oc
y
yL
0.248λ
yL' = 1-j2.9
0.2985λ

[B] A coaxial line operates at 2 GHz and is designed to have an impedance of 50Ω. Assume that the coaxial line is filled with dielectric material whose relative permittivity ϵ_r is 2.25, has an inner copper conductor diameter is 2mm.

1. Derive the expression for capacitance per unit length using Gaus's Law.
2. Derive the expression for the inductance using Ampere's Law.
3. What is inner diameter of the outer conductor to ensure the impedance is 50Ω.
4. What is the phase velocity of this coaxial line?

Capacitance

Gaussian surface at radius r. Assume inner voltage is V, outer = 0V



ϵ_r Inner radius a, outer b.

Assume charge per unit length on inner is Q

Flux density $\bar{a}_r D_r$

Then $2\pi r \cdot 1 \cdot D_r = Q$

$$\bar{a}_r D_r = \frac{Q}{2\pi r}$$

$$\bar{a}_r E_r = \frac{Q}{2\pi \epsilon_0 \epsilon_r r}$$

$$V = \int_b^a \bar{a}_r E_r \cdot dr = \frac{Q}{2\pi\epsilon_0\epsilon_r} \ln\left(\frac{b}{a}\right) \qquad C = \frac{Q}{V} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)} \quad \text{— (1)}$$

Similarly $2\pi r \cdot H_r = I$, $H_r = \frac{I}{2\pi r}$, $B_r = \frac{\mu_0 I}{2\pi r}$

$$\text{Total flux } \Phi / m \text{ (radial flux)} = \int_a^b \frac{\mu_0 I}{2\pi r} dr = \frac{\mu_0 I}{2\pi} \ln(b/a) \quad \text{— (2)}$$

$$L = \frac{\Phi}{I} = \frac{\mu_0}{2\pi} \ln(b/a)$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0}{2\pi} \ln(b/a) \cdot \frac{\ln(b/a)}{2\pi\epsilon_0\epsilon_r}} \qquad \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0 = 377\Omega.$$

$$= \ln\left(\frac{b}{a}\right)\frac{1}{2\pi}\sqrt{\frac{\mu_0}{\epsilon_0}\frac{1}{\epsilon_r}}$$

$$= \ln(b/a)\frac{377}{1.5}\times\frac{1}{2\pi} = 50, \quad \ln(b/a) = \frac{1.5\times 50\times 2\pi}{377} = 1.25$$

$$b/a = e^{1.25} = 3.49$$

Hence $b = 3.49$mm, diameter $= 6.98$mm — (3)

$$v_{ph} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s}$$